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PARAMETRIC INSTABILITY IN THE OSCILLATIONS OF A BODY MOVING UNIFORMLY
IN A PERIODICALLY INHOMOGENEOUS ELASTIC SYSTEM

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UDC 531.534

Transitional radiation of various types occurs in the uniform and rectilinear motion of a perturbation source in an inhomogeneous medium. In [1, 2], there is a survey of such radiation for electromagnetic and acoustic waves. In [3], the radiation was examined for elastic waves arising in the uniform motion of a mechanical object in an inhomogeneous elastic system. Features of the radiation in such a system are related to the interaction between the radiation and the oscillations of the object. Here we consider parametric object oscillation excitation during emission.

When a perturbing source moves in a periodically inhomogeneous medium, the radiation has a discrete spectrum in the steady state [1]. In a reference system coupled to the moving source, the spectrum is equidistant. The object moving uniformly in a periodically inhomogeneous elastic system is subject to a transverse force equivalent to the reaction of a spring with periodically varying rigidity. That situation naturally leads to parametric object oscillation [4], which is demonstrated here. It is necessary to examine such interaction for example in relation to the requirements for high-speed railroad transportation. A train moving over rails under certain conditions may begin to show a galloping motion, and here we show that the parameter region where it occurs expands as the speed increases.

1. We consider the uniform motion $z = vt$ of a body with mass m along an unbounded string whose tension and density per unit length are correspondingly N and ρ , and which lies on a periodically inhomogeneous elastic base. The rigidity of the base is described by

$$k(z) = k_0(1 + \mu \cos(2\pi z/d)),$$

in which k_0 is the mean rigidity, d the period of the inhomogeneity, and $\mu \ll 1$ a dimensionless small parameter.

A description of the self-consistent motion of the body and string is [5]

$$\begin{aligned} U_{tt} - U_{xx} + U(1 + \mu \cos(\kappa x)) &= 0, \quad U(\alpha t, t) = y(t), \\ (1 - \alpha^2) [U_x]_{x=\alpha t} &= M\ddot{y}(t), \quad [U]_{x=\alpha t} = 0, \quad U \rightarrow 0 \text{ for } x \rightarrow \pm\infty. \end{aligned} \quad (1.1)$$

Here $U(x, t)$ is the transverse deviation of the string, $x = zh/c$ and $t = ht$ ($c^2 = N/\rho$, $h^2 = k_0/\rho$) are the dimensionless coordinate and time, $y(t)$ the transverse coordinate of the body, with $M = mh/c\rho$ and $\alpha = v/c$ (with $\alpha < 1$ subsequently) the dimensionless mass and longitudinal velocity, and $\kappa = 2\pi c/dh$. The square brackets denote the differences between the values of the expressions in them to the right and left of the given x .

We seek the solution to (1.1) as

$$U = U^0 + \mu U^1 + \dots, \quad y = y^0 + \mu y^1 + \dots \quad (1.2)$$

2. In the zeroth approximation ($\mu = 0$), (1.2) represents the motion of a body on a string lying on a homogeneous elastic base:

$$\begin{aligned} U_{tt}^0 - U_{xx}^0 + U^0 &= 0, \quad U^0(\alpha t, t) = y^0(t), \\ (1 - \alpha^2) [U_x^0]_{x=\alpha t} &= M\ddot{y}^0(t), \quad [U^0]_{x=\alpha t} = 0, \quad U^0 \rightarrow 0 \text{ for } x \rightarrow \pm\infty. \end{aligned} \quad (2.1)$$

As the solution to (2.1), one naturally takes a function describing the oscillation of the body-string system for $t \rightarrow \infty$. We first determine the oscillation frequency for $t \rightarrow \infty$.

Nizhnii Novgorod. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 2, pp. 127-133, March-April, 1993. Original article submitted January 4, 1992.

We apply an integral Fourier transformation with respect to the coordinate to (2.1):

$$V^0(k, t) = \int_{-\infty}^{\infty} U^0(x, t) \exp(ikx) dx.$$

In terms of transforms we have

$$V_{tt}^0 + (1 + k^2)V^0 = -M\ddot{y}(t) \exp(iakt). \quad (2.2)$$

We take the initial conditions as zero (this is permissible because we are interested only in the natural frequencies of the body at this stage), and write the solution to (2.2) as a convolution:

$$V^0(k, t) = -M \int_0^t \ddot{y}^0(\tau) \exp(iakt) \frac{\sin((t-\tau)\sqrt{1+k^2})}{\sqrt{1+k^2}} d\tau. \quad (2.3)$$

We apply an inverse Fourier transformation to (2.3), reverse the integration order, and use the formula [5]

$$\frac{1}{\pi} \int_0^{\infty} \cos(kf) \frac{\sin(a\sqrt{b^2+k^2})}{\sqrt{b^2+k^2}} dk = \frac{1}{2} J_0(b\sqrt{a^2-f^2}) \theta(a-|f|)$$

in which θ is unit function and J_0 a zero-order Bessel function to get

$$U^0(x, t) = -\frac{M}{2} \int_0^t \ddot{y}^0(\tau) J_0(\sqrt{(t-\tau)^2 - (x-\alpha\tau)^2}) \theta((t-\tau) - |x-\alpha\tau|) d\tau. \quad (2.4)$$

We use the condition that the body moves without detachment and the features of the convolution operation to get from (2.4) an equation describing the oscillations (we recall that $\alpha < 1$):

$$y^0(t) + \frac{M}{2} \int_0^t \ddot{y}^0(t-\tau) J_0(\tau\sqrt{1-\alpha^2}) d\tau = 0. \quad (2.5)$$

We substitute $y^0(t) = A \exp(i\Omega t)$ into (2.5) and take $t \rightarrow \infty$ to get an equation defining the natural frequencies of the body for $t \rightarrow \infty$:

$$1 - M\Omega^2/2\sqrt{1-\alpha^2-\Omega^2} = 0,$$

whence

$$\Omega = \pm \frac{\sqrt{2}}{M} \sqrt{\sqrt{1+M^2(1-\alpha^2)} - 1}.$$

The oscillations for $t \rightarrow \infty$ will thus be described here by

$$y^0(t) = A \exp(i\Omega t) + B \exp(-i\Omega t). \quad (2.6)$$

The oscillations of the string corresponding to the steady-state oscillations of the body for $t \rightarrow \infty$ are readily derived by substituting (2.6) into (2.1) and seeking the solution as

$$U(x, t) = \sum_n C_n \exp(i\omega_n t - ik_n x). \quad (2.7)$$

The solution is completely analogous to that derived in [6, 7], so we write the final expression for $U^0(x, t)$ at once:

$$\begin{aligned} U^0 &= C_A \exp(i\omega_j t - ik_j x) + C_B \exp(i\omega_{j+2} t - ik_{j+2} x), \\ j &= \begin{cases} \text{II} & x < \alpha t, \\ \text{I} & x > \alpha t, \end{cases} \\ k_{1,2} &= (\alpha\Omega \pm i\sqrt{1-\alpha^2-\Omega^2})/(1-\alpha^2), \quad \omega_{1,2} = \alpha k_{1,2} + \Omega, \\ k_{3,4} &= (-\alpha\Omega \pm i\sqrt{1-\alpha^2-\Omega^2})/(1-\alpha^2), \quad \omega_{3,4} = \alpha k_{3,4} - \Omega, \\ C_A &= A\Omega^2/2\sqrt{1-\alpha^2-\Omega^2}, \quad C_B = B\Omega^2/2\sqrt{1-\alpha^2-\Omega^2}. \end{aligned} \quad (2.8)$$

Then for $t \rightarrow \infty$, a body moving uniformly over a homogeneous string and oscillating with frequency Ω is accompanied by deflection in the string, whose oscillation amplitude falls away from the body.

3. We now consider the first approximation with respect to the small parameter μ . We substitute (1.2) into (1.1) and equate terms of order μ to get

$$U_{tt}^1 - U_{xx}^1 + U^1 = -U^0 \cos(\alpha x), \quad U^1(\alpha t, t) = y^1(t), \quad (3.1)$$

$$(1 - \alpha^2) [U_x^1]_{x=\alpha t} = M \ddot{y}^1(t), \quad [U^1]_{x=\alpha t} = 0, \quad U^1 \rightarrow 0 \quad \text{for } x \rightarrow \pm \infty.$$

The solution to (3.1) is a superposition of the forced solution for the first equation, which defines the oscillation frequency for the moving mass, and the solution due to those oscillations. We seek the solution to the first equation in (3.1) in the (2.7) form:

$$U_{\text{for}}^1 = C_1^j \exp(i\omega_j t - ix(k_j + \alpha)) + C_2^j \exp(i\omega_j t - ix(k_j - \alpha)) + C_3^j \exp(i\omega_{j+2} t - ix(k_{j+2} - \alpha)) + C_4^j \exp(i\omega_{j+2} t - ix(k_{j+2} + \alpha)), \quad (3.2)$$

in which

$$C_{1,2}^j = -C_A/2k_j(k_j \pm 2\alpha); \quad C_{3,4}^j = -C_B/2k_{j+2}(k_{j+2} \pm 2\alpha).$$

The solution to (3.2) defines the oscillation frequency for the moving mass because the phases of the waves propagating in the string at $x = \alpha t$ should coincide with those of the mass oscillation. One uses this and writes for $y^1(t)$ that

$$y^1(t) = \sum_{m=1}^2 \{A_m \exp(it(\Omega + (-1)^m \alpha \kappa)) + A_{m+2} \exp(-it(\Omega + (-1)^m \alpha \kappa))\}. \quad (3.3)$$

The oscillations in the moving mass in turn generate corresponding oscillations in the string. We substitute (3.3) into (3.1) and define the solution as the (2.7) traveling waves to get a solution describing the oscillations of the string excited by the uniformly moving mass oscillating in accordance with (3.3):

$$U_{\text{nat}}^1 = D_1^j \exp(i\omega_j^1 t - ik_j^1 x) + D_2^j \exp(i\omega_{j+2}^1 t - ik_{j+2}^1 x) + D_3^j \exp(i\omega_{j+4}^1 t - ik_{j+4}^1 x) + D_4^j \exp(i\omega_{j+6}^1 t - ik_{j+6}^1 x). \quad (3.4)$$

Here

$$k_{1,2}^1 = (\alpha(\Omega - \alpha \kappa) \pm i\sqrt{1 - \alpha^2 - (\Omega - \alpha \kappa)^2}) / (1 - \alpha^2), \quad \omega_{1,2} = \alpha(k_{1,2} - \alpha) + \Omega;$$

$$k_{3,4}^1 = (\alpha(\Omega + \alpha \kappa) \pm i\sqrt{1 - \alpha^2 - (\Omega + \alpha \kappa)^2}) / (1 - \alpha^2), \quad \omega_{3,4} = \alpha(k_{3,4} + \alpha) + \Omega;$$

$$k_{5,6}^1 = (-\alpha(\Omega + \alpha \kappa) \pm i\sqrt{1 - \alpha^2 - (\Omega + \alpha \kappa)^2}) / (1 - \alpha^2), \quad \omega_{5,6} = \alpha(k_{5,6} - \alpha) - \Omega;$$

$$k_{7,8}^1 = (-\alpha(\Omega - \alpha \kappa) \pm i\sqrt{1 - \alpha^2 - (\Omega - \alpha \kappa)^2}) / (1 - \alpha^2), \quad \omega_{7,8} = \alpha(k_{7,8} + \alpha) - \Omega.$$

To derive the unknowns A_n and D_n ($n = 1, 4$), it is necessary to use the nondetachment condition for a body $U^1(\alpha t, t) = y^1(t)$, the condition for the continuity of the string at the point where the mass is $[U^1]_{x=\alpha t} = 0$, and the balance for the transverse forces on the moving mass $(1 - \alpha^2)[U_x^1]_{x=\alpha t} = M \ddot{y}^1(t)$. As $U^1 = U_{\text{for}}^1 + U_{\text{nat}}^1$, we get

$$A_n = \{(\sqrt{1 - \alpha^2 - (\Omega + (-1)^n \alpha \kappa)^2} - \sqrt{1 - \alpha^2 - \Omega^2})(C_n^I + C_n^{II}) - (-1)^n i \alpha (C_n^I - C_n^{II})\} / \{2\sqrt{1 - \alpha^2 - (\Omega + (-1)^n \alpha \kappa)^2} - M(\Omega + (-1)^n \alpha \kappa)^2\}$$

$$D_n = A_n - C_n^I. \quad (3.5)$$

Then (2.8), (3.2), and (3.4) describe the oscillations of the string and correspondingly of the body for $t \rightarrow \infty$ if $\alpha \kappa$ is not close to twice the natural frequency of the oscillations for the body moving over a homogeneous string Ω . In the resonant case, when $2\Omega = \pm \alpha \kappa$, A_n and D_n tend to infinity, which is readily seen from (3.5), on the basis that Ω satisfies $2 \times \sqrt{1 - \alpha^2 - \Omega^2} - M\Omega^2 = 0$.

When the system parameters satisfy

$$2\Omega = \pm \alpha \kappa + \delta_1 \quad (3.6)$$

($\delta_1 \ll \Omega$, small frequency difference), the solution is incorrect; (3.6) is analogous to the condition for parametric resonance in a system described by a Mathieu equation [4]:

$$\ddot{x} + \omega_0^2(1 + \cos(\omega_p t)) = 0.$$

In fact, Ω is the natural frequency of a body moving uniformly on a uniform string and corresponds to the natural frequency ω_0 in the unperturbed Mathieu treatment, while $\alpha \kappa$ is simply the frequency of change in ω_p , since the rigidity of the elastic base changes with frequency $\alpha \kappa$ under the mass uniformly moving with velocity α .

Then (3.6) is analogous to the condition $2\omega_0 = \omega_p + \delta_1$, under which classical parametric resonance sets in (basic instability zone).

4. We consider the resonant case, where

$$\kappa = (2\Omega + \mu\delta)/\alpha. \quad (4.1)$$

The solution to (1.1) on the parametric resonance scheme used in [4] is sought as

$$y(t) = A(\tau)\exp(it(\Omega + \mu\delta)) + B(\tau)\exp(-it(\Omega + \mu\delta)) + \mu y^1(t), \\ U(x, t) = C_1^j(\tau, \xi)\exp(it(\omega_j + \mu\delta) - ik_j x) + C_2^j(\tau, \xi)\exp(it(\omega_{j+2} - \mu\delta) - ik_{j+2} x) + \mu U^1(x, t), \quad (4.2)$$

in which $\tau = \mu t$; $\xi = \mu x$; ω_n and k_n ($n = 1, 4$) are defined by (2.8).

We substitute (4.2) into the first equation in (1.1) to get to terms of order μ that

$$U_{tt}^1 - U_{xx}^1 + U^1 = -2\exp(it(\omega_j + \mu\delta) - ik_j x) \{ \omega_j (iC_{1\tau}^j - \delta C_1^j) + \\ + ik_j C_{1\xi}^j \} - 2\exp(it(\omega_{j+2} - \mu\delta) - ik_{j+2} x) \{ \omega_{j+2} (iC_{2\tau}^j + \delta C_2^j) + ik_{j+2} C_{2\xi}^j \} - \\ - \cos(\kappa x) (C_1^j \exp(it(\omega_j + \mu\delta) - ik_j x) + C_2^j \exp(it(\omega_{j+2} - \mu\delta) - ik_{j+2} x)). \quad (4.3)$$

For $U^1(x, t)$ not to increase with time, (4.3) shows that the following conditions must be met:

$$\omega_j (iC_{1\tau}^j - \delta C_1^j) + ik_j C_{1\xi}^j = 0, \quad \omega_{j+2} (iC_{2\tau}^j + \delta C_2^j) + ik_{j+2} C_{2\xi}^j = 0. \quad (4.4)$$

We assume that (4.4) is met and discard those terms on the right in (4.3) whose phases for $x = \alpha t$ are different from $it(\Omega - \alpha\kappa)$ and $-it(\Omega - \alpha\kappa)$ [or on the basis of (4.1), from $it(\Omega + \mu\delta)$ and $-it(\Omega + \mu\delta)$] to get from (4.3) that

$$U_{tt}^1 - U_{xx}^1 + U^1 = -\frac{1}{2} C_1^j \exp(it(\omega_j + \mu\delta) - i(k_j + \kappa)x) - \frac{1}{2} C_2^j \exp(it(\omega_{j+2} - \mu\delta) - i(k_{j+2} - \kappa)x). \quad (4.5)$$

We write the solution to (4.5), as in the nonresonant case, as a superposition of the forced and natural solutions:

$$U^1 = U_{\text{for}}^1 + U_{\text{nat}}^1. \quad (4.6)$$

Here $U_{\text{for}}^1 = W_1^j \exp(it(\omega_j + \mu\delta) - i(k_j + \kappa)x) + W_2^j \exp(it(\omega_{j+2} - \mu\delta) - i(k_{j+2} - \kappa)x)$;

$$W_{1,2}^j = -C_{1,2}^j / \{2\kappa(\kappa \pm 2k_{j,j+2})\}.$$

We now examine the balance in the transverse forces acting on the moving mass. We use

$$(1 - \alpha^2) [U_x]_{x=\alpha t} = M\ddot{y}(t),$$

and write the $U(x, t)$ and $y(t)$ written in accordance with (1.1) into (4.2) and restrict consideration to terms of order μ and use (4.6) to get

$$(1 - \alpha^2) [U_{x\text{nat}}^1]_{x=\alpha t} - M\ddot{y}^1(t) = \exp(it(\Omega + \mu\delta)) \{ 2\Omega M (i\dot{A} - \delta A) - \\ - (1 - \alpha^2) (C_{1\xi}^{\text{II}} - C_{1\xi}^{\text{I}} - i(k_1 - \kappa)W_2^{\text{II}} + i(k_3 - \kappa)W_2^{\text{I}}) \} + \exp(-it(\Omega + \mu\delta)) \times \\ \times \{ -2\Omega M (i\dot{B} + \delta B) - (1 - \alpha^2) (C_{2\xi}^{\text{II}} - C_{2\xi}^{\text{I}} - i(k_2 + \kappa)W_1^{\text{II}} + i(k_1 + \kappa)W_1^{\text{I}}) \}. \quad (4.7)$$

The natural frequency of the mass is Ω , so for $y^1(t)$ not to increase in time, the expressions in braces in (4.7) should become zero, i.e.,

$$2\Omega M (i\dot{A} - \delta A) - (1 - \alpha^2) (C_{1\xi}^{\text{II}} - C_{1\xi}^{\text{I}} - i(k_1 - \kappa)W_2^{\text{II}} + i(k_3 - \kappa)W_2^{\text{I}}) = 0, \\ -2\Omega M (i\dot{B} + \delta B) - (1 - \alpha^2) (C_{2\xi}^{\text{II}} - C_{2\xi}^{\text{I}} - i(k_2 + \kappa)W_1^{\text{II}} + i(k_1 + \kappa)W_1^{\text{I}}) = 0. \quad (4.8)$$

Then (4.4) and (4.8) represent the necessary conditions for $U^1(x, t)$ and $y^1(t)$ not to increase. We seek the solutions to (4.4) and (4.8) as

$$A(\tau) = A_0 \exp(s\tau), \quad B(\tau) = B_0 \exp(s\tau), \\ C_1^j = C_{10}^j \exp(\lambda_1^j \tau - q_1^j \xi), \quad C_2^j = C_{20}^j \exp(\lambda_2^j \tau - q_2^j \xi)$$

and use the condition for nondetachment and for continuity of the string, which imply

$$C_{10} = C_{10} = A_0, \quad C_{20} = C_{20} = B_0, \quad \lambda_{1,2} - \alpha q_{1,2} = s,$$

to get after certain algebraic steps that (4.4) and (4.8) imply

$$\begin{aligned} \frac{2\Omega M}{1-\alpha^2}(is-\delta)A_0 - \frac{4(\delta-is)}{M\Omega(1-\alpha^2)}A_0 + \frac{M\Omega^2\alpha^2}{8(\alpha^2+\Omega^2)}B_0 &= 0, \\ \frac{2\Omega M}{1-\alpha^2}(is+\delta)B_0 + \frac{4(\delta+is)}{M\Omega(1-\alpha^2)}B_0 - \frac{M\Omega^2\alpha^2}{8(\alpha^2+\Omega^2)}A_0 &= 0. \end{aligned} \quad (4.9)$$

From (4.9), we get a characteristic equation for s :

$$s^2 = -\delta^2 + \frac{\alpha^4\Omega^2(1-\alpha^2)^2}{4^4(\alpha^2+\Omega^2)^2(1+2/M^2\Omega^2)^2}.$$

The condition for unstable body oscillation (parametric resonance) is that s is real. Then from (4.1) the bounds to the instability region ($s = 0$) are defined by

$$\alpha\kappa - 2\Omega \pm \mu \frac{\alpha^2\Omega(1-\alpha^2)^2}{16(\alpha^2+\Omega^2)(1+2/M^2\Omega^2)} = 0. \quad (4.10)$$

Figure 1 shows the hatched instability zones in the α, M plane derived from (4.10) for various κ (it is assumed that $\mu = 0.1$). The following conclusions are drawn from Fig. 1:

- the less the inhomogeneity period d (the larger κ), the lower the α at which instability sets in;
- as the speed increases, the mass of the body at which parametric resonance occurs decreases; and
- as the mass of the body and/or the inhomogeneity period increase, the instability zone expands in α .

The uniform motion of a body in a periodically inhomogeneous elastic system causes a parametric increase in the body oscillation amplitude. We thus have to consider the origin of the energy for pumping the oscillations. It is sufficient to recall that transitional radiation occurs in the uniform motion of a body in an inhomogeneous elastic system, which exerts a pressure on the body [3]. Consequently, to maintain the uniform motion, one needs an external source, whose work goes to increase the energy of the oscillations in the body for the corresponding parameters.

The (4.10) bounds to the instability zone represent the bounds to the basic instability zone obtained in the first approximation with respect to μ . To determine the bounds to the subsequent instability regions, which arise for $\alpha\kappa = 2\Omega/n + \delta_1$ ($n = 1, 2, \dots$), or the bounds to the main instability zone with higher accuracy, one needs to change the form of the (4.2) solution by analogy with what was done in [4].

5. We have to consider how minor viscosity in the elastic base affects the result. The first equation in (1.1) is rewritten as follows on the basis of a small dimensionless viscosity $2\mu\nu$ in the base:

$$U_{tt} - U_{xx} + 2\mu\nu U_t + U(1 + \mu \cos(\kappa x)) = 0. \quad (5.1)$$

We substitute the (4.1) solution into (5.1) and take steps analogous to those in Sec. 4 to get the condition on the parameters for instability to occur:

$$\frac{M^2\Omega^4(1-\alpha^2)^2}{16(\alpha^2+\Omega^2)^2(\nu^2+16\Omega^2/\alpha^4)} - \delta^2(M+2/M\Omega^2)^2 - \frac{4\nu^2}{M^2\Omega^4} > 0. \quad (5.2)$$

This shows that when one incorporates low viscosity in the base ν , a region appears in the α, M plane even for zero δ ($2\Omega = \alpha\kappa$) in which the oscillations of the body are stable for any κ . We put $\delta = 0$ in (5.2) to get the threshold viscosity:

$$\nu_{\text{thr}} = 2\Omega\alpha^2 \left(-2 + 2 \sqrt{1 + \frac{M^2\Omega^4(1-\alpha^2)^2}{64^2\alpha^8(\alpha^2+\Omega^2)^2}} \right)^{1/2}.$$

Figure 2 shows lines for ν_{thr} in the α, M plane. Each curve separates the plane into two regions. In the region above the curve, there is always a κ for which there is instability, while in the region below the curve, the oscillations of the body are stable for any κ . Figure 2 also shows that:

- for a given speed, the threshold viscosity decreases as the mass of the body falls (the oscillations may become unstable at a lower base viscosity); and
- with a given body mass, the threshold viscosity is the lower the higher the velocity.

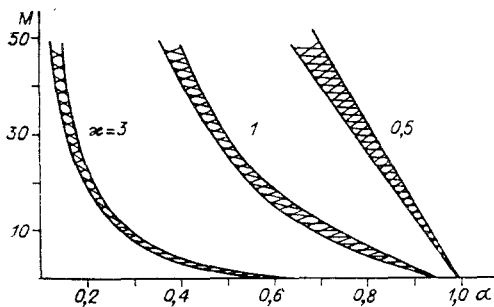


Fig. 1

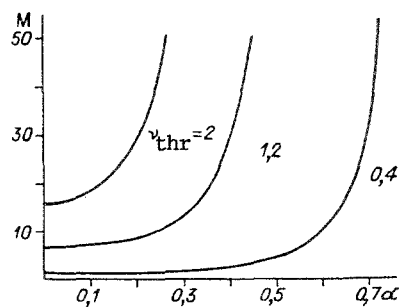


Fig. 2

We thus get parametric resonance in this uniform motion, which is seen as an increase in the amplitude of the body oscillations that is exponential for $t \rightarrow \infty$. The work needed to increase the energy in the body oscillations is performed by the external source that maintains the uniform motion.

Here we have considered nondetachment motion. In a real situation such as a train moving on rails, the wheels may separate from the rails and then renew contact with shock interaction. Consequently, galloping can occur, which involves a sequence of contact breaks and renewals. If the parameters considered here belong to the stability region, it does not mean that the motion of the body is free from detachment. If the parameters fall in the region of instability, contact is bound to be broken for $t \rightarrow \infty$, i.e., it is the sufficient condition for galloping.

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